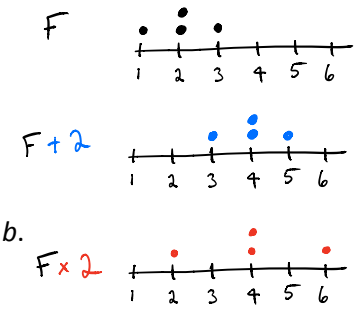
Statistics – Ch 6.2 Notes **Name:** Key

Transforming and Combining Random Variables

Remember from Chapter 2:

1. **Adding (or subtracting) a constant, a, to each observation:**
   - Adds *a* to measures of center and location.
   - Does not change the shape or measures of spread.
2. **Multiplying (or dividing) each observation by a constant, b:**
   - Multiplies (divides) measures of center and location by *b*.
   - Multiplies (divides) measures of spread by |b|.
   - Does not change the shape of the distribution.



Location
- percentiles
- quartiles
- min
- max

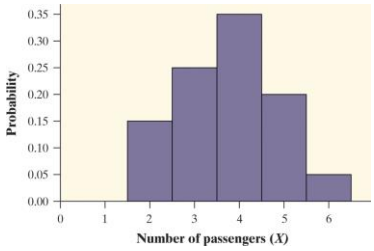
Center
- mean
- median

Spread
- Range
- Standard dev.
- IQR
- Variance

Multiplying a Random Variable by a Constant

1. Pete's Jeep Tours offers a popular half-day trip in a tourist area. There must be at least 2 passengers for the trip to run, and the vehicle will hold up to 6 passengers. Define *X* as the number of passengers on a randomly selected day.

Passengers x_i	2	3	4	5	6
Probability p_i	0.15	0.25	0.35	0.20	0.05



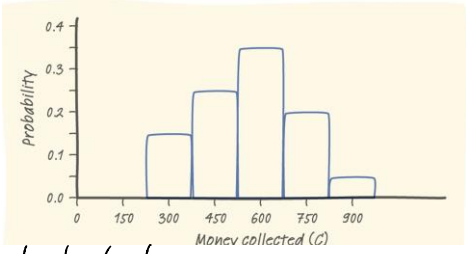
a) Define the Random variable X and find its mean and standard deviation.

X = number of passengers on a randomly selected day

$\mu_x = 3.75$ $\sigma_x = 1.0897$

2. Suppose Pete charges $150 per passenger. The random variable *R* describes the revenue Pete collects on a randomly selected day.

Revenue	300	450	600	750	900
Probability	0.15	0.25	0.35	0.20	0.05



b) Define the Random variable R and find its mean and standard deviation.

R = revenue collected on a randomly selected day.

$R = 150X$

c) Find the mean and standard deviation of the random variable R.

$\mu_R = 150\mu_x$

$\mu_R = 150(3.75)$

$\boxed{\mu_R = \$562.50}$

$\sigma_R = 150\sigma_x$

$\sigma_R = 150(1.0897)$

$\boxed{\sigma_R = \$163.455}$

Effect on a Random Variable of Multiplying (Dividing) by a Constant

Multiplying (or dividing) each value of a random variable by a number b:
- Multiplies (divides) measures of center and location (mean, median, quartiles, percentiles) by *b.*
- Multiplies (divides) measures of spread (range, *IQR*, standard deviation) by |*b*|.
- Does not change the shape of the distribution.

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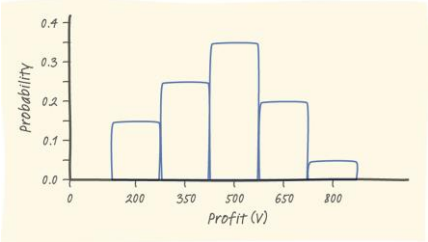
Transforming and Combining Random Variables

Adding a Constant to a Random Variable

3. It costs Pete $100 per trip to buy permits, gas, and a ferry pass. The random variable P describes the profit Pete makes on a randomly selected day.

Profit	200	350	500	650	800
Probability	0.15	0.25	0.35	0.20	0.05



a) Define the Random variable P and find its mean and standard deviation.

P = Profit on a randomly selected day

$P = R - 100$

b) Find the mean and standard deviation of the random variable P.

$\mu_P = \mu_R - 100$

$\mu_P = 562.50 - 100$

$\boxed{\mu_P = \$462.50}$

$\sigma_P = \sigma_R$

$\boxed{\sigma_P = \$163.455}$

Effect on a Random Variable of Adding (or Subtracting) a Constant

Adding the same number a (which could be negative) to each value of a random variable:

- Adds a to measures of center and location (mean, median, quartiles, percentiles).
- Does not change measures of spread (range, IQR, standard deviation).
- Does not change the shape of the distribution.

skip

Independent Practice

A large auto dealership keeps track of sales made during each hour of the day. Let X = the number of cars sold during the first hour of business on a randomly selected Friday. Based on previous records, the probability distribution of X is as follows.

Cars Sold	0	1	2	3
Probability	0.3	0.4	0.2	0.1

μ_X = 1.1 Cars

σ_X = 0.943

1. Suppose the dealership's manager receives a $500 bonus from the company for each car sold. Let **Y = the bonus received from car sales during the first hour on a randomly selected Friday.** Find the mean and standard deviation of Y.

$Y = 500X$

$\mu_y = 500\mu_x$

$\mu_y = 500(1.1)$

$\boxed{\mu_y = \$550}$

$\sigma_y = 500\sigma_x$

$\sigma_y = 500(0.943)$

$\boxed{\sigma_y = \$471.50}$

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Transforming and Combining Random Variables

SKIP

Independent Practice

A large auto dealership keeps track of sales made during each hour of the day. Let X = the number of cars sold during the first hour of business on a randomly selected Friday. Based on previous records, the probability distribution of X is as follows.

Bonus	0	$500	$1000	$1500
Probability	0.3	0.4	0.2	0.1

$\mu_Y = \$550$
$\sigma_Y = \$471.50$

2. To encourage customers to buy cars on Friday mornings, the manager spends $75 to provide coffee and doughnuts. The manager's net profit **T** on a randomly selected Friday is the bonus minus this $75. Find the mean and standard deviation of **T.**

$T = Y - 75$

$\mu_T = \mu_Y - 75$

$\mu_T = 550 - 75$

$\boxed{\mu_T = \$475}$

$\sigma_T = \sigma_Y$

$\boxed{\sigma_T = \$471.5}$

Effect on a Linear Transformation on the Mean and Standard Deviation

If $Y = a + bX$ is a linear transformation of the random variable X, then

- The probability distribution of Y has the same shape as the probability distribution of X.
- $\mu_Y = a + b\mu_X$.
- $\sigma_Y = |b|\sigma_X$ (since b could be a negative number).

We could have gone directly from the number of passengers X on a randomly selected jeep tour to Pete's profit P with the equation

P = 150X – 100 Doesn't Change Spread

This linear transformation includes both:

1. Multiplying by 150 and
2. Subtracting 100

$\mu_P = 150\mu_X - 100$

$\mu_P = 150(3.75) - 100$

$\boxed{\mu_P = \$462.50}$

$\sigma_P = 150\sigma_X$

$\sigma_P = 150(1.0897)$

$\boxed{\sigma_P = \$163.455}$

What is the net effect of this sequence of transformations?

- **Shape:** neither changes the shape of the probability distribution.
- **Center:** the mean of X is multiplied by 150 then decreased by 100;
  $\mu_Y = 150\mu_X -- 100$
- **Spread:** the standard deviation of X is multiplied by 150 and is unchanged by the subtraction:
  $\sigma_Y = 150\sigma_X$

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Transforming and Combining Random Variables

Independent Practice

One brand of bathtub comes with a dial to set the water temperature. When the "baby safe" setting is selected and the tub is filled, the temperature X of the water follows a Normal distribution with mean 34 degrees Celsius and a standard deviation of 2 degrees Celsius.

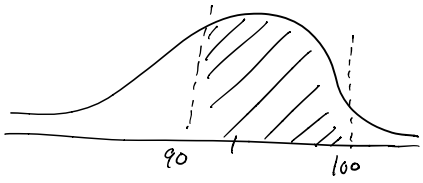
a) Define the random variable Y to be the water temperature in degrees Fahrenheit (recall that F = 9/5C + 32) when the dial is set on "baby safe". Find the mean and standard deviation.

$Y = \frac{9}{5}X + 32$ $\mu_y = \frac{9}{5}(34) + 32$ $\sigma_y = \frac{9}{5}(2)$

$\boxed{\mu_y = 93.2\,°F}$ $\boxed{\sigma_y = 3.6\,°F}$

b) According to Babies R Us, the temperature of a baby's bathwater should be between 90 degrees Fahrenheit and 100 degrees Fahrenheit. Find the probability that the water temperature on a randomly selected day when the "baby safe" setting is used meets the Babies R Us recommendation Show your work.

$N(93.2, 3.6)$



$P(90 \le Y \le 100) = \text{Normalcdf}(90, 100, 93.2, 3.6)$

$P(90 \le Y \le 100) = 0.7835$

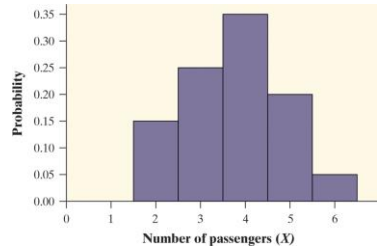
The probability that the bath water temperature on a randomly selected day, when using the "baby safe" setting, meets the Babies R Us recommendation is 0.7835.

Statistics – Ch 6.2 Notes **Name:** Key

Transforming and Combining Random Variables

Combing Random Variables

Earlier, we examined the probability distribution for the random variable X = the number of passengers on a randomly selected half-day trip with Pete's Jeep Tours.



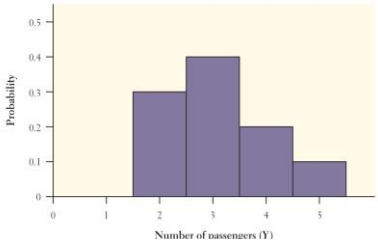
Passengers x_i	2	3	4	5	6
Probability p_i	0.15	0.25	0.35	0.20	0.05

X = number of Pete's passengers on a randomly selected day

$\mu_X = 3.75$

$\sigma_X = \cancel{1.090}$ 1.0897

Pete's sister is impressed and decides to join the business, running tours on the same days as Pete an a completely different part of the country. Erin discovers that the number of passengers Y on her half-day tours has the following probability distribution.



Passengers Y	2	3	4	5
P(Y)	0.3	0.4	0.2	0.1

Y = number of Erin's passengers on a randomly selected day

$\mu_Y = 3.1$

$\sigma_Y = 0.943$

<u>Combining Random Variables (Sum):</u>

1. Define a new Random Variable ***T = the total number of passengers Pete and Erin will have on their tours on a randomly selected day.***
   a. Create an equation to represent the random variable T.

   $T = X + Y$

   b. Find the mean of the distribution of T.

   $\mu_T = \mu_X + \mu_Y$

   $\mu_T = 3.75 + 3.1$

   $\boxed{\mu_T = 6.85 \text{ passengers}}$

   a. Find the standard deviation of the distribution of T.

   $\sigma_T^2 = \sigma_X^2 + \sigma_Y^2$

   $\sigma_T^2 = (1.0897)^2 + (0.943)^2$

   $\sigma_T^2 = 2.0767$

   $\sigma_T = \sqrt{2.0767}$

   $\boxed{\sigma_T = 1.441}$

   * only if random variables are independent
   * must find variance first

Independent Practice:

2. A college uses SAT scores as one criterion for admission. Experience has shown that the distribution of SAT scores among its entire population of applicants is such that:

SAT Math score X:	$\mu_X = 519$	$\sigma_X = 115$
SAT Critical Reading Score Y:	$\mu_Y = 507$	$\sigma_Y = 111$

   a) What is the mean of the total score T = X + Y for a randomly selected applicant to this college?

   $\mu_T = 519 + 507$

   $\boxed{\mu_T = 1026}$

   b) What is the mean of the total score T = X + Y for a randomly selected applicant to this college?

   Variance and standard deviation cannot be computed b/c math scores and reading scores are not independent. Students who score high on one exam tend to score high on the other also.

Statistics – Ch 6.2 Notes **Name:** Key

Transforming and Combining Random Variables

<u>Combining Random Variables with Linear Transformations</u>

3. Earlier we defined **X = the number of passengers that Pete has** and **Y = the number of passengers that Erin has** on a randomly selected day. Pete charges $150 per passenger and Erin charges $175 per passenger.

$$\mu_x = 3.75 \quad \sigma_x = 1.0897$$
$$\mu_Y = 3.10 \quad \sigma_Y = 0.943$$

Calculate the mean and standard deviation of the total amount that Pete and Erin collect on a randomly chosen day.

C = Pete's $ collected → $C = 150X$

G = Erin's $ collected → $G = 175Y$

W = Total amount collected → $W = C + G$

C

$\mu_c = 150(3.75)$

$\mu_c = 562.5$

$\sigma_c = 150(1.0897)$

$\sigma_c = 163.46$

G

$\mu_G = 175(3.10)$

$\mu_G = 542.5$

$\sigma_G = 175(0.943)$

$\sigma_G = 165.03$

W

$\mu_W = \mu_c + \mu_G$

$\mu_W = 562.5 + 542.6$

$\mu_W = 1105$ on average Pete and Erin expect to collect $1105 per day

$\sigma_W^2 = \sigma_c^2 + \sigma_G^2$

$\sigma_W^2 = (163.46)^2 + (165.03)^2$

$\sqrt{\sigma_W^2} = \sqrt{53,954.07}$

$\sigma_W = 232.28$

Independent Practice:

4. A large auto dealership keeps track of sales and lease agreements made during each hour of the day. Let X = the number of cars sold and Y = the number of cars leased during the first hour of business on a randomly selected Friday. Based on previous records, the probability distribution of X and Y are as follows.

Cars sold x_i:	0	1	2	3
Probability p_i:	0.3	0.4	0.2	0.1

Mean: $\mu_X = 1.1$ Standard deviation: $\sigma_X = 0.943$

Cars leased y_i:	0	1	2
Probability p_i:	0.4	0.5	0.1

Mean: $\mu_Y = 0.7$ Standard deviation: $\sigma_Y = 0.64$

Define T = X + Y. Assume that X and Y are independent.

a) Find and interpret μ_T

$\mu_T = 1.1 + 0.7$

$\mu_T = 1.8$

over many fridays, this dealership sells or leases about 1.8 cars in the first hour of business on average.

b) Compute σ_T. Show your work.

$\sigma_T^2 = (0.943)^2 + (0.64)^2$

$\sqrt{\sigma_T^2} = \sqrt{1.2988}$

$\sigma_T^2 = 1.1396$

Transforming and Combining Random Variables

c) The dealership's manager receives a $500 bonus for each car sold and $300 bonus for each car leased. Find the mean and standard deviation of the manager's bonus B. Show your work.

S = Cars Sold bonus → $S = 500X$

L = Cars Leased bonus → $L = 300Y$

B = Total Bonus → $B = S + L$

S

$\mu_S = 500(1.1)$

$\mu_S = 550$

$\sigma_S = 500(.943)$

$\sigma_S = 471.5$

L

$\mu_L = 300(0.7)$

$\mu_L = 210$

$\sigma_L = 300(0.64)$

$\sigma_L = 192$

B

$\mu_B = \mu_S + \mu_L$

$\mu_B = 550 + 210$

$\boxed{\mu_B = \$760}$

$\sigma_B^2 = \sigma_S^2 + \sigma_L^2$

$\sigma_B^2 = (471.5)^2 + (192)^2$

$\sqrt{\sigma_B^2} = \sqrt{259,176.25}$

$\boxed{\sigma_B = \$509.09}$

<u>Combining Random Variables (Difference):</u>

5. Define a new random variable ***D = the difference between the number of Pete's passengers and Erin's passengers.***

C = amount of money that Pete collects
G = amount of money that Erin collects

Here are the means and standard deviations of these random variables:

$\mu_C = 562.50$	$\mu_G = 542.50$
$\sigma_C = 163.46$	$\sigma_G = 165.03$

PROBLEM: Calculate the mean and the standard deviation of the difference ***D* = *C* – G** in the amounts that Pete and Erin collect on a randomly chosen day. Interpret each value in context.

$D = C - G$

$\mu_D = \mu_C - \mu_G$

$\mu_D = 562.5 - 542.5$

$\boxed{\mu_D = \$20}$

$\sigma_D^2 = \sigma_C^2 + \sigma_G^2$

$\sigma_D^2 = (163.46)^2 + (165.03)^2$

$\sqrt{\sigma_D^2} = \sqrt{53,954.0725}$

$\boxed{\sigma_D = \$232.28}$

on average, Pete collects $20 more per day than Erin does. Even though the average difference in the amount collected is $20, the difference on individual days will typically vary from the mean by about $232.28

Transforming and Combining Random Variables

Independent Practice

6. A large auto dealership keeps track of sales and lease agreements made during each hour of the day. Let X = the number of cars sold and Y = the number of cars leased during the first hour of business on a randomly selected Friday. Based on previous records, the probability distributions of X and Y are as follows:

Cars sold x_i:	0	1	2	3
Probability p_i:	0.3	0.4	0.2	0.1

Mean: $\mu_X = 1.1$ Standard deviation: $\sigma_X = 0.943$

Cars leased y_i:	0	1	2
Probability p_i:	0.4	0.5	0.1

Mean: $\mu_Y = 0.7$ Standard deviation: $\sigma_Y = 0.64$

Define $D = X - Y$. Assume that X and Y are independent.

a) Find and interpret μ_D.

$\mu_D = 1.1 - 0.7$

$\mu_D = 0.4$

over many days this dealership sells about 0.4 cars more than it leases, during the first hour of business.

b) Compute σ_D. Show your work.

$\sigma_D = \sqrt{(0.943)^2 + (0.64)^2}$

$\sigma_D = 1.1396$

c) The dealership's manager receives a \$500 bonus for each car sold and a \$300 bonus for each car leased. Find the mean and standard deviation of the difference in the manager's bonus for cars sold and leased. Show your work.

S = Cars Sold Bonus

L = Cars Leased Bonus

D = Difference between Bonus

S

$\mu_S = 550$

$\sigma_S = 471.5$

L

$\mu_L = 210$

$\sigma_L = 192$

D

$\mu_D = \mu_S - \mu_L$

$\mu_D = 550 - 210$

* $\boxed{\mu_D = \$340}$

$\sqrt{\sigma_D^2} = \sqrt{(471.5)^2 + (192)^2}$

* $\boxed{\sigma_D = 509.09}$

Transforming and Combining Random Variables

<u>Example: Sums of Normal Random Variables</u>

Mr. Starnes likes sugar in his hot tea. From experience, he needs between 8.5 and 9 grams of sugar in a cup of tea for the drink to taste right. While making his tea one morning, Mr. Starnes adds four randomly selected packets of sugar. Suppose the amount of sugar in these packets follows a Normal distribution with mean 2.17 grams and standard deviation 0.08 grams.

PROBLEM: What's the probability that Mr. Starnes's tea tastes right?

<u>State</u>: What is the probability that Mr. Starnes tea tastes right?

X = amount of sugar in a randomly selected packet

X_1 = amount in 1st packet

X_2 = amount in 2nd Packet

X_3 = amount in 3rd Packet

X_4 = amount in 4th Packet

$\mu_X = 2.17$

$\sigma_X = 0.08$

$T = X_1 + X_2 + X_3 + X_4$

$\mu_T = 2.17 + 2.17 + 2.17 + 2.17$

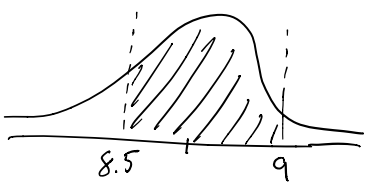
$\mu_T = 8.68$ grams

$\sigma_T^2 = (0.08)^2 + (0.08)^2 + (0.08)^2 + (0.08)^2$

$\sqrt{\sigma_T^2} = \sqrt{0.0256}$

$\sigma_T = 0.16$ grams

<u>Perform</u>:



normalcdf(8.5, 9, 8.68, 0.16) = 0.8470

<u>Conclude</u>:

There is about an 85% chance that Mr. Starnes's tea will tast right.

In Conclusion: $X_1 + X_2 \neq 2X$

- For situations when an individual observation is doubled use: $2X$
- For situations involving repeated observations from the same chance process. $X_1 + X_2$

Transforming and Combining Random Variables

Key Concepts and Formulas:

1. Mean of the Sum of Random Variables

For any two random variables X and Y, if $T = X + Y$, then the expected value of T is

$$E(T) = \mu_T = \mu_X + \mu_Y$$

In general, the mean of the sum of several random variables is the sum of their means.

2. Independent random variables.

If knowing whether any event involving X alone has occurred tells us nothing about the occurrence of any event involving Y alone, and vice versa, then X and Y are **independent random variables**.

In our investigation, it is reasonable to assume X and Y are independent since the siblings operate their tours in different parts of the country.

3. Variance of the Sum of Random Variables

For any two *independent* random variables X and Y, if $T = X + Y$, then the variance of T is

$$\sigma_T^2 = \sigma_X^2 + \sigma_Y^2$$

In general, the variance of the sum of several independent random variables is the sum of their variances.

****What does this mean?****

Remember that you can add variances only if the two random variables are independent, *and that you can NEVER add* Standard Deviations!